

Find the equations of all horizontal asymptotes of $f(x) = \frac{2e^{7x} + 3}{15 - 8e^{7x}}$.

SCORE: ____ / 15 PTS

$$\lim_{x \rightarrow \infty} \frac{2e^{7x} + 3}{15 - 8e^{7x}} \quad \frac{\infty}{\infty}$$

$$= \lim_{x \rightarrow \infty} \frac{2 + 3e^{-7x}}{15e^{-7x} - 8}$$

$$= \frac{2 + 3(0)}{15(0) - 8}$$

$$= -\frac{1}{4}$$

$$\lim_{x \rightarrow -\infty} \frac{2e^{7x} + 3}{15 - 8e^{7x}}$$

$$= \frac{2(0) + 3}{15 - 8(0)}$$

$$= \frac{1}{5}$$

$$y = -\frac{1}{4}, y = \frac{1}{5}$$

If $\lim_{x \rightarrow 3} \frac{f(x)+7}{x-3} = -2$, find $\lim_{x \rightarrow 3} f(x)$ and prove that your answer is correct.

SCORE: ____ / 15 PTS

$$\lim_{x \rightarrow 3} \frac{f(x)+7}{x-3} \cdot \lim_{x \rightarrow 3} (x-3) = -2 \cdot \lim_{x \rightarrow 3} (x-3)$$

SINCE $\lim_{x \rightarrow 3} (x-3)$ EXISTS (EQUALS 0)

$$\lim_{x \rightarrow 3} \frac{f(x)+7}{x-3} \cdot (x-3) = 0$$

SINCE $\lim_{x \rightarrow 3} \frac{f(x)+7}{x-3}$ ALSO EXISTS (GIVEN)

$$\lim_{x \rightarrow 3} (f(x)+7) = 0$$

$$\lim_{x \rightarrow 3} (f(x)+7) + \lim_{x \rightarrow 3} (-7) = \lim_{x \rightarrow 3} (-7)$$

SINCE $\lim_{x \rightarrow 3} (-7)$ EXISTS (EQUALS -7)

$$\lim_{x \rightarrow 3} (f(x)+7-7) = -7 \quad \text{ie.} \quad \lim_{x \rightarrow 3} f(x) = -7$$

SINCE $\lim_{x \rightarrow 3} (f(x)+7)$ ALSO EXISTS (SHOWN ABOVE)

At time t minutes, the position of an object moving in a straight line is $p(t) = \frac{5t-9}{3+t}$ yards.

SCORE: ____ / 25 PTS

- [a] Find the average velocity of the object from time $t = 1$ to time $t = 5$. Give the units of your answer.

$$\frac{p(5) - p(1)}{5 - 1} = \frac{2 - (-1)}{4} = \frac{3}{4} \text{ YARDS/MINUTE}$$

- [b] Find the instantaneous velocity of the object at time $t = 2$. Give the units of your answer.

$$\lim_{b \rightarrow 2} \frac{\frac{5b-9}{3+b} - \frac{1}{5}}{b-2}$$

$$= \lim_{b \rightarrow 2} \frac{5(5b-9) - (3+b)}{5(b-2)(3+b)}$$

$$= \lim_{b \rightarrow 2} \frac{\cancel{24b} - 48 + 24}{5(b-2)(3+b)} = \frac{24}{25} \text{ YARDS/MINUTE}$$

State the formal definition of "removable discontinuity".

SCORE: ____ / 5 PTS

f HAS A REMOVABLE DISCONTINUITY AT a IF $\lim_{x \rightarrow a} f(x)$ EXISTS BUT
EITHER $f(a)$ DOES NOT EXIST OR $\lim_{x \rightarrow a} f(x) \neq f(a)$

State the Squeeze Theorem.

SCORE: ____ / 10 PTS

IF $f(x) \leq g(x) \leq h(x)$ FOR ALL x NEAR a (EXCEPT POSSIBLY AT a)

AND $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$

THEN $\lim_{x \rightarrow a} g(x) = L$

Prove that the equation $x = 3 \ln x$ has a solution in the interval $(1, e)$.

SCORE: ____ / 15 PTS

$$\text{LET } f(x) = x - 3 \ln x$$

f IS CONT. ON $(1, e)$ SINCE x AND $3 \ln x$ ARE BOTH CONT. ON $(1, e)$
(POLY + LOG)

SO THEIR DIFFERENCE IS ALSO CONT.

$$f(e) = e - 3 < 0 < 1 = f(1)$$

BY IVT, THERE IS A VALUE $c \in (1, e)$

SUCH THAT $f(c) = 0$

$$\text{IE. } c - 3 \ln c = 0$$

$$c = 3 \ln c$$

SO $x = 3 \ln x$ FOR SOME $x \in (1, e)$

Pat's score on today's midterm depends on how much sleep Pat got in the last 24 hours.

SCORE: _____ / 15 PTS

Let $m = g(s)$, where m is the number of points Pat scores on the midterm, and s is the number of hours Pat slept in the last 24 hours.

- [a] What are the units of $g'(s)$?

POINTS / HOUR

- [b] What does $g'(5) = 11$ mean ? Give the units for all numbers in your answer.

IF PAT HAD SLEPT FOR 5 OF THE LAST 24 HOURS BEFORE THE MIDTERM, PAT'S SCORE WOULD HAVE INCREASED ≈ 11 POINTS FOR EACH ADDITIONAL HOUR OF SLEEP PAT HAD GOTTEN.

- [c] Is there a value of s_0 for which you would expect $g'(s_0) < 0$? Why or why not ?

YES, IF PAT SLEPT TOO MUCH, THEN PAT STUDIED LESS OR COULD HAVE ENDED UP GROGGY, AND MORE SLEEP WOULD CAUSE A DROP IN PAT'S SCORE

Let $f(x) = \frac{1}{\sqrt{x}}$.

SCORE: ____ / 30 PTS

[a] Find $f'(x)$.

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{x} - (x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \\ &= \lim_{h \rightarrow 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \\ &= \frac{-1}{2x\sqrt{x}} \text{ OR } -\frac{1}{2}x^{-\frac{3}{2}} \end{aligned}$$

[b] Find the equation of the tangent to the graph of f at the point where $x = 9$.

$$f(9) = \frac{1}{3} \quad f'(9) = -\frac{1}{54}$$

$$y - \frac{1}{3} = -\frac{1}{54}(x - 9)$$

Let $f(x) = \frac{x-1}{x(x+3)^2}$.

SCORE: ____ / 20 PTS

- [a] Find all intervals on which f is continuous.

$$x \neq 0, -3$$

$$(-\infty, -3), (-3, 0), (0, \infty)$$

- [b] Find the limit of f at each discontinuity.

Each limit should be a number, ∞ or $-\infty$. Write DNE only if the other possibilities do not apply.

$$\lim_{x \rightarrow 0^+} \frac{x-1}{x(x+3)^2} = -\infty$$

$$\frac{-1}{0^+}$$

$$\lim_{x \rightarrow -3^+} \frac{x-1}{x(x+3)^2} = \infty$$

$$\frac{-4}{0^+}$$

$$\lim_{x \rightarrow 0^-} \frac{x-1}{x(x+3)^2} = \infty$$

$$\frac{-1}{0^-}$$

$$\lim_{x \rightarrow -3^-} \frac{x-1}{x(x+3)^2} = \infty$$

$$\frac{-4}{0^-}$$

$$\lim_{x \rightarrow 0} f(x) \text{ DNE}$$

$$\lim_{x \rightarrow -3} f(x) = \infty$$